Sculpture, Geometry and Computer Science

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SIAM CSE 2011
Feb 28 – Mar 4
Reno, NV

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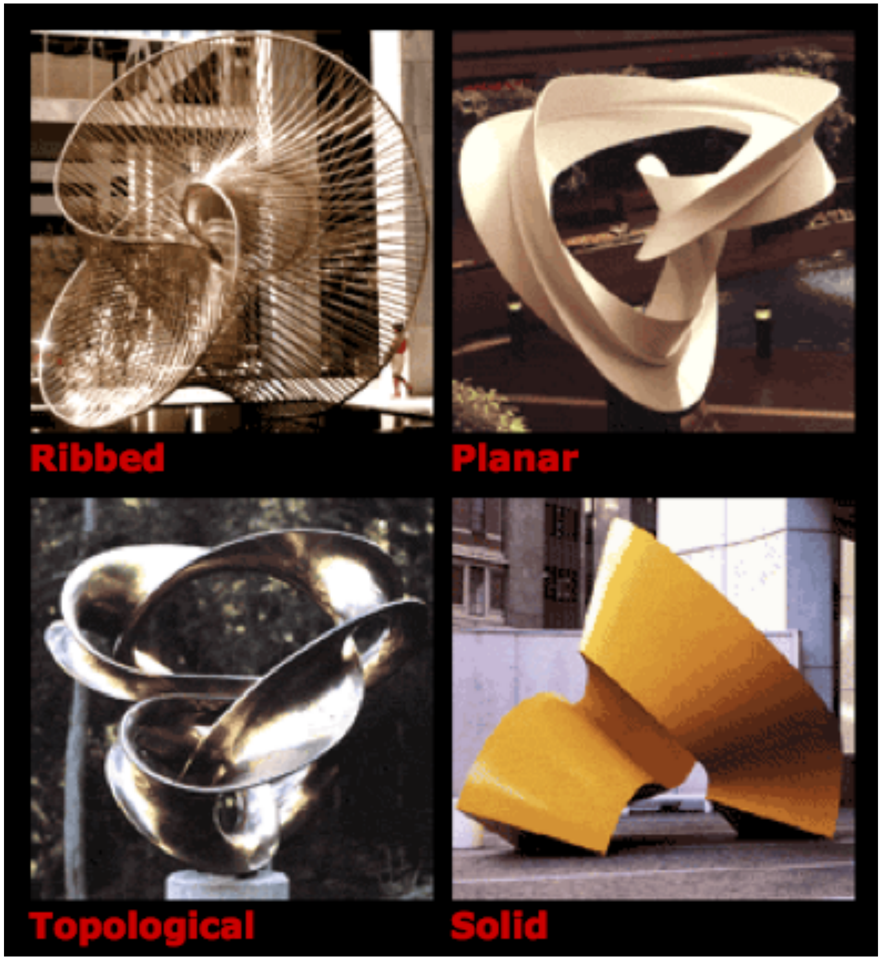
Overview: Project for MS/HS students

- Observation and Appreciation (of sculpture)
- Modeling goal: create a polygonal model on computers
  - Measure: exact, estimate
  - Problem solving: approximation, divide & conquer
  - Programming, Interactive Graphics
  - Mathematics
    - Analytic Geometry, Trigonometry
    - Symmetry, Matrix Multiplication
- Homage to "Geometry and the Imagination"
- An alternative to gaming

Geometric Sculpture (Charles Perry)



“Continuum”
National Air and Space Museum.

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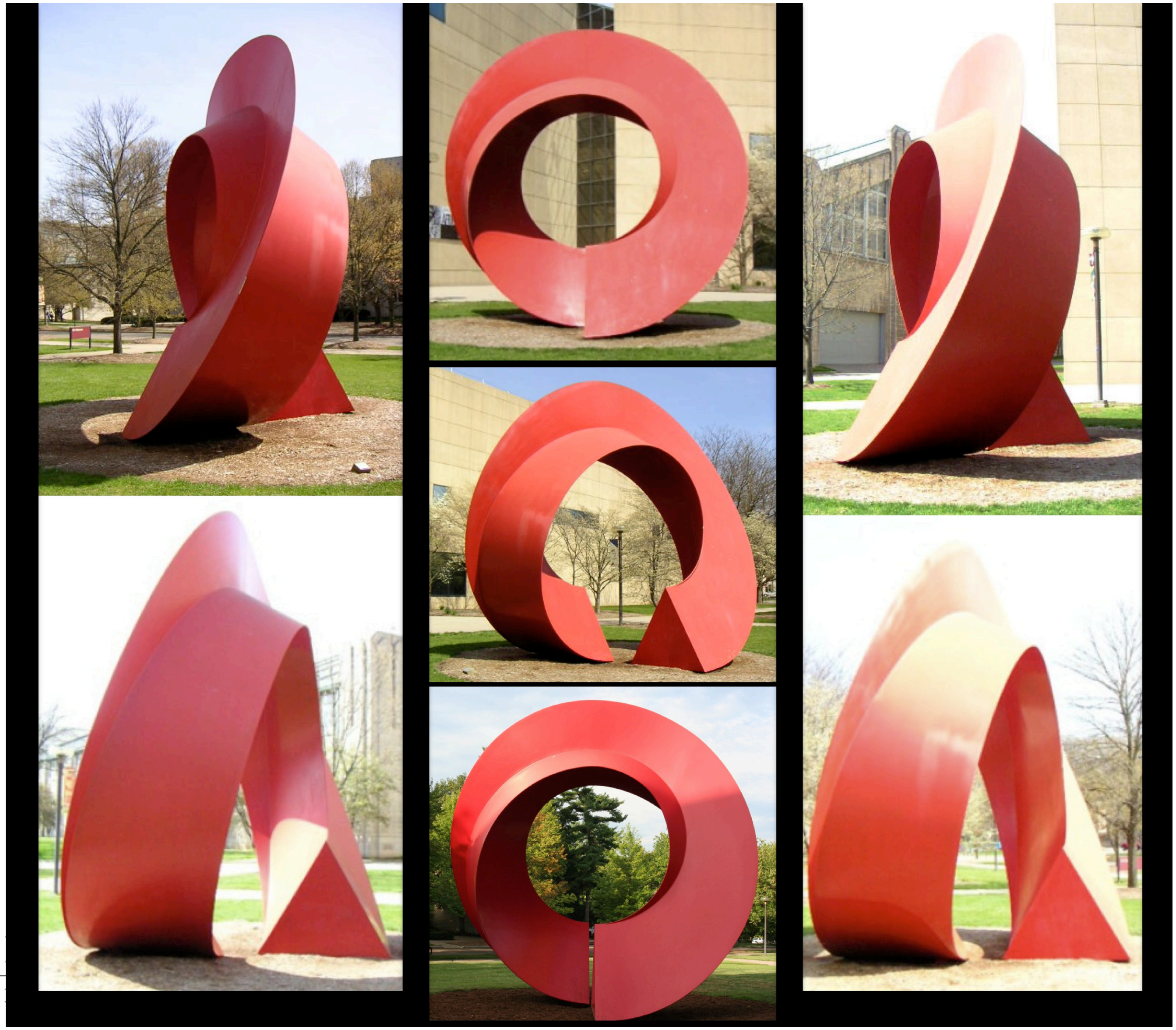
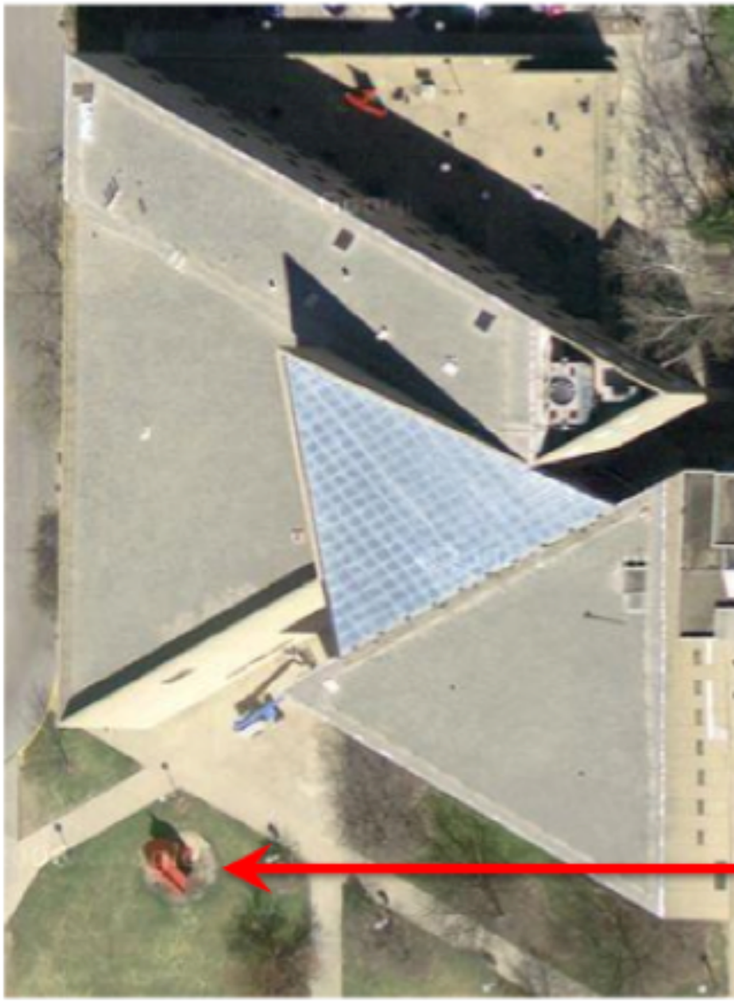
Indiana Arc: Indiana University campus (IU Art Museum)

W



E

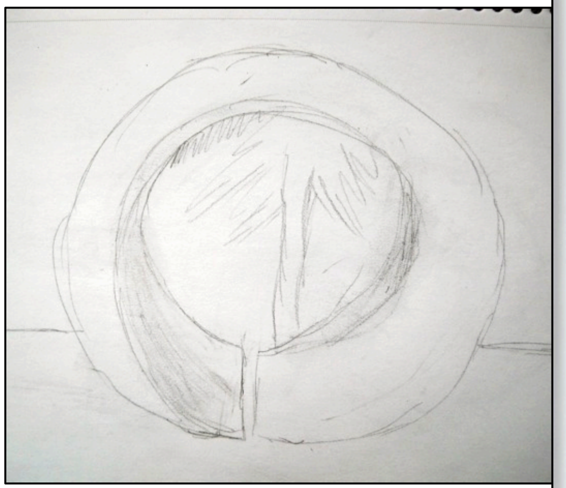
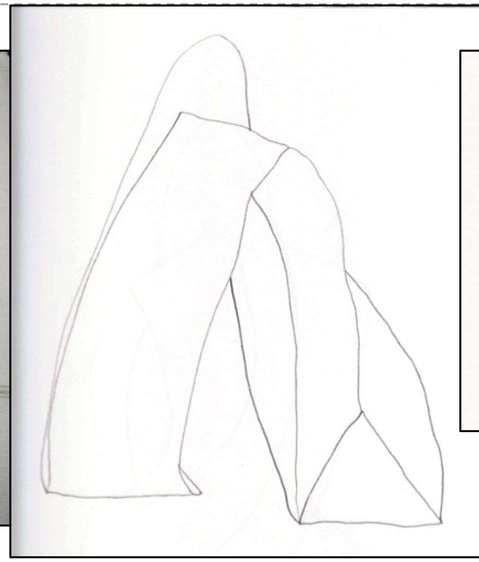
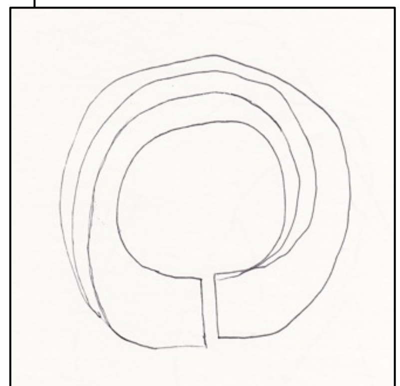
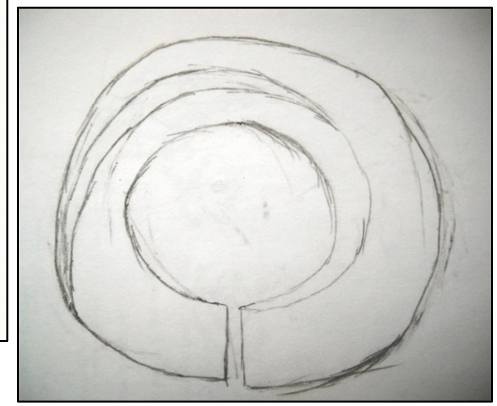
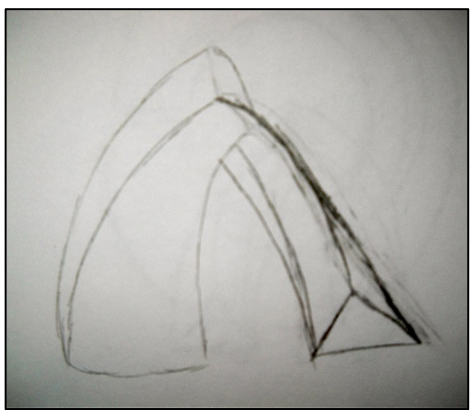
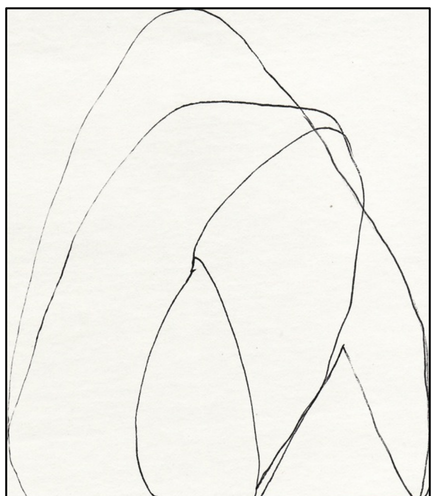
Backdrop: IU Art Museum - I.M. Pei



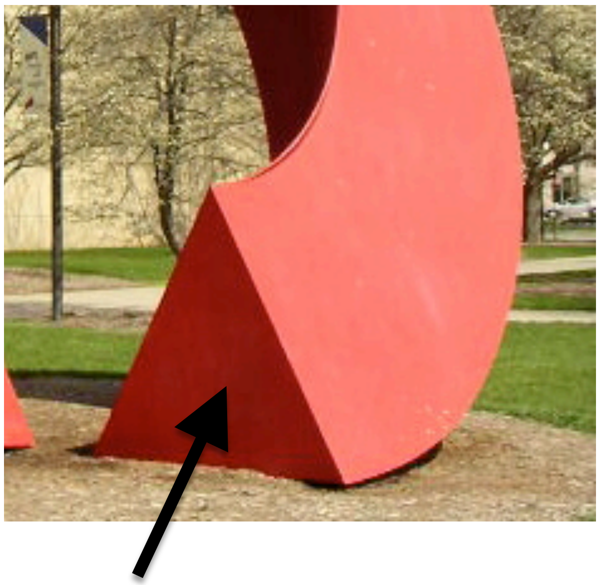
Observation & Appreciation

Allowed sketchbooks

Sketches

Measure: Exact and Estimated



Equilateral triangle:
184 cm sides

height = 184 * sin 60
~= 159

Estimate height, radii

Modeling – how to begin (and disclaimer)

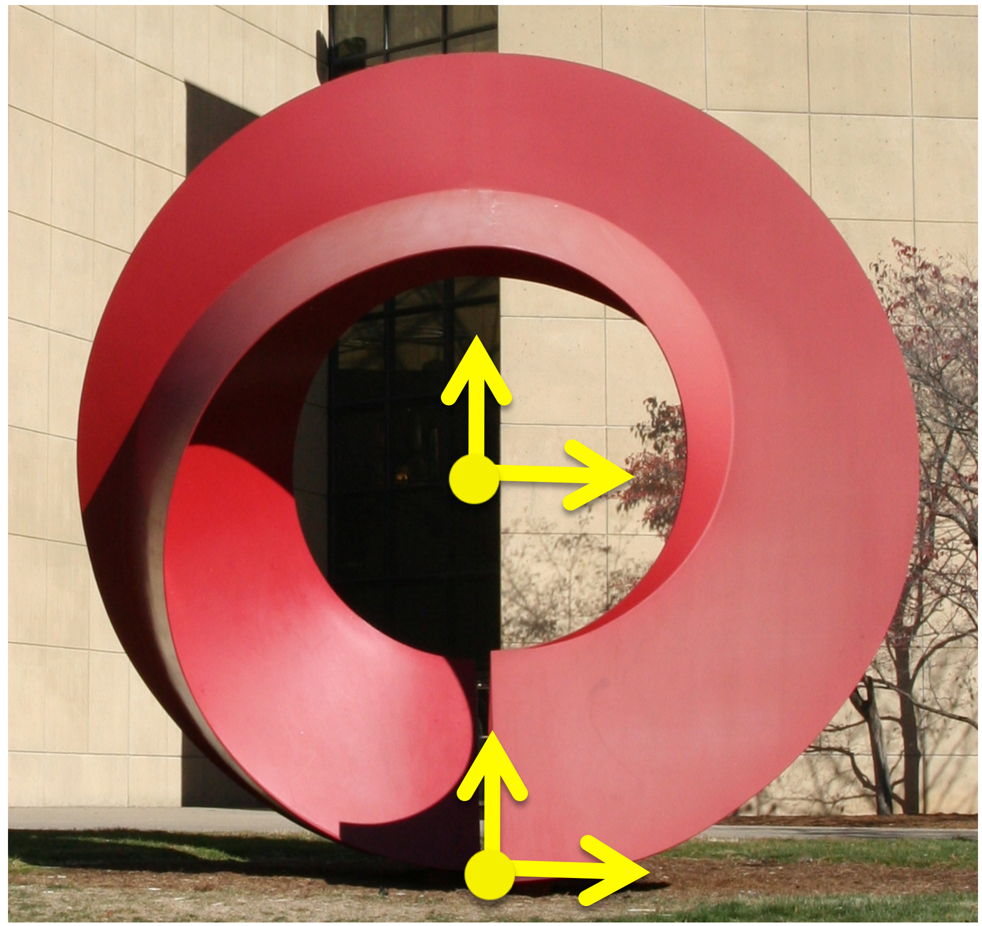
We do not claim to **accurately** reproduce this sculpture's shape, but approximate it – as is often the case for models in computational science.

Geometry and the Imagination

- David Hilbert and Stefan Cohn-Vossen (1932)
- Want to give students a small sense of math history
- "…it is still as true today as it ever was that *intuitive* understanding plays a major role in geometry."
- "…generally speaking, mathematics is not a popular subject. The reason for this is the common superstition that math is just a continuation of arithmetic." They aim to combat that with "figures, instead of formulas that the reader may easily construct".
- "We want to take the reader on a leisurely walk…in the big garden that is geometry, so that each may pick a bouquet to his liking."

http://en.wikipedia.org/wiki/David_Hilbert

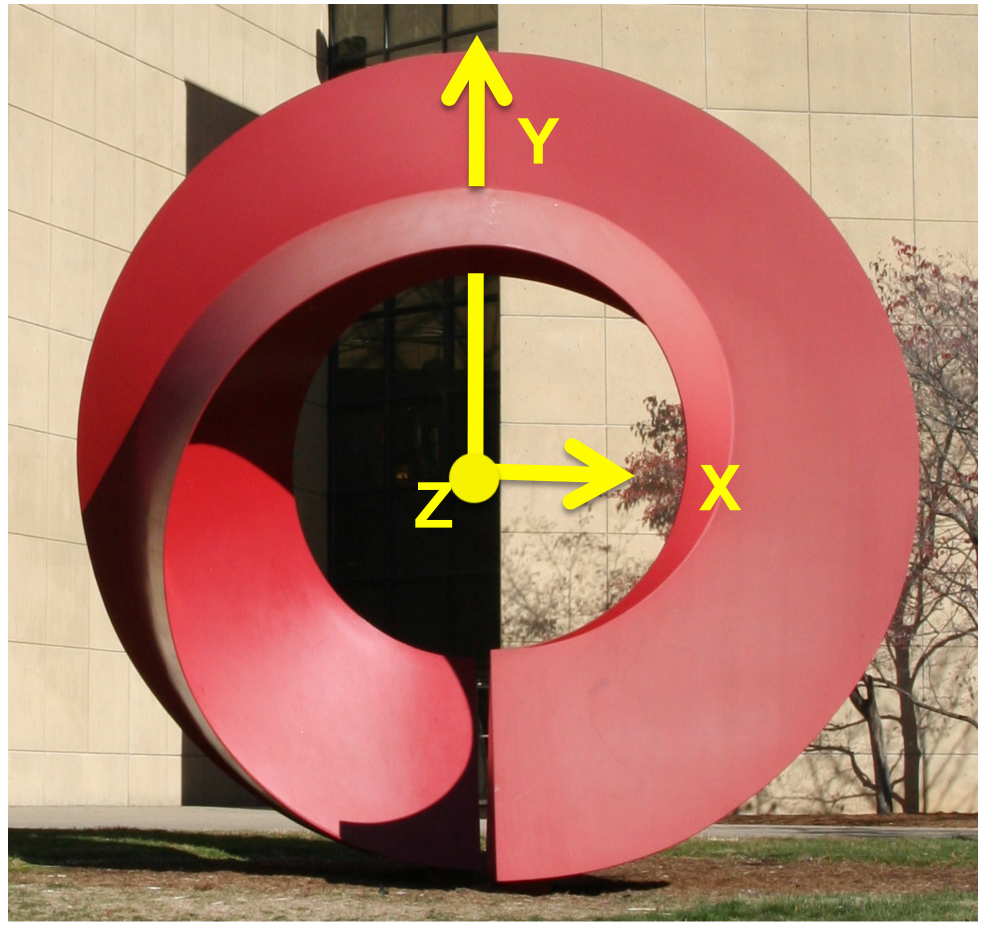
Choosing a coordinate system(s)



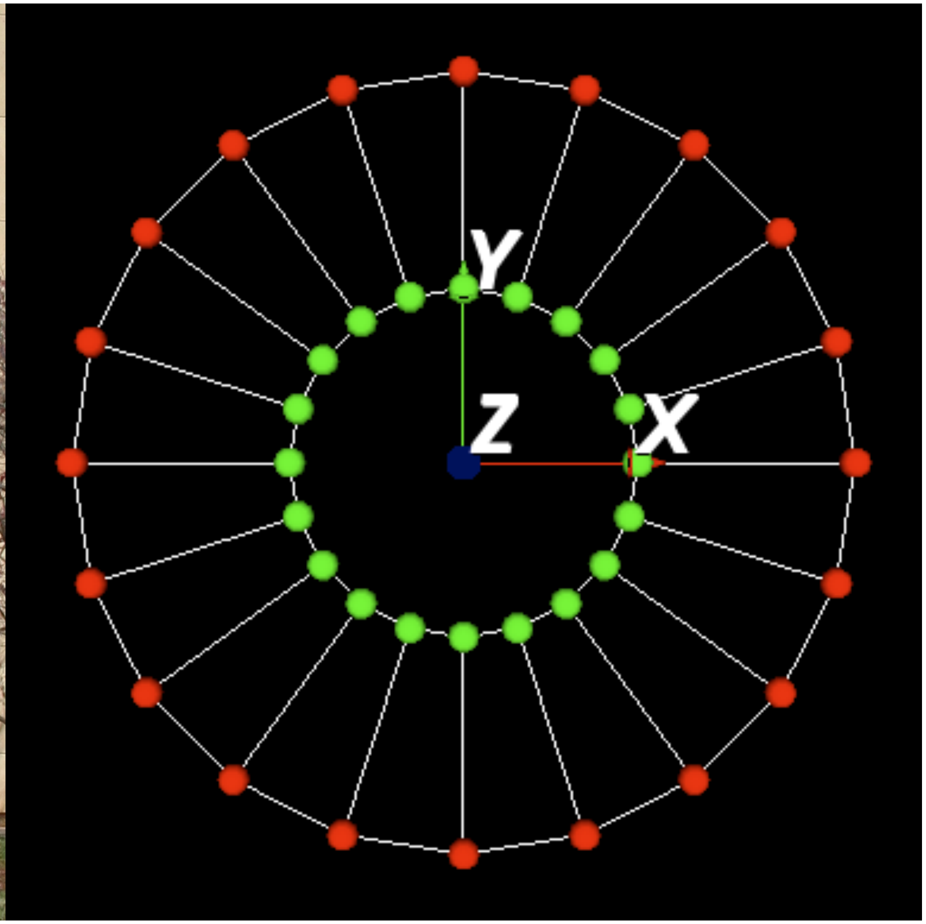
Is there a "best" coordinate system?

Coordinate System

Perhaps this is the most logical choice. Let's try it.



Approximation #1: an Annulus (2D)



Leads to programming…

Python (programming) and VTK (graphics)

- Install Python and IPython
- Install Notepad++ editor (students used Windows)
- Install binary VTK (for Python) [and matplotlib]

- Python programming 101: (mention students' experience)

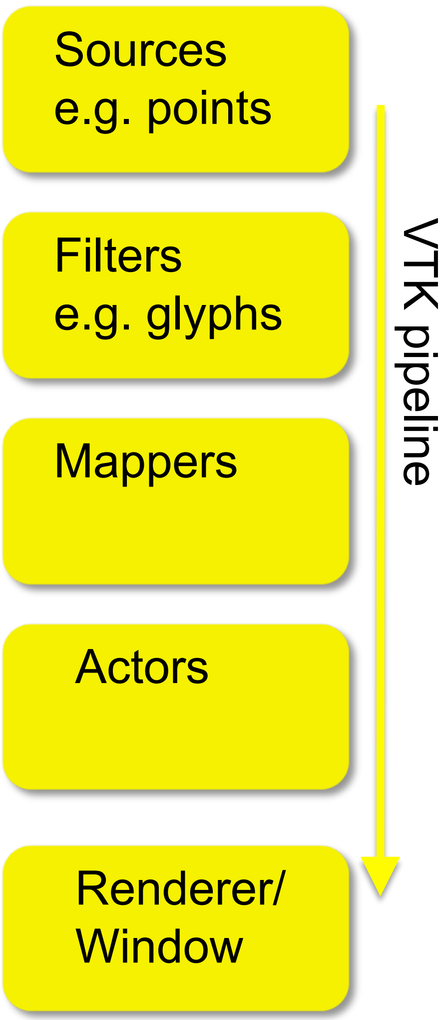
"=" is assignment not equality:

```
>>> x = 13
>>> x = x + 1
>>> print x
14
```

```
from math import *

Loops:
For i in range(0,npts):
   x = Ro * cos(theta)
   y = Ro * sin(theta)
   …
```

VTK basics (for polygonal model)

(we provide a template script)

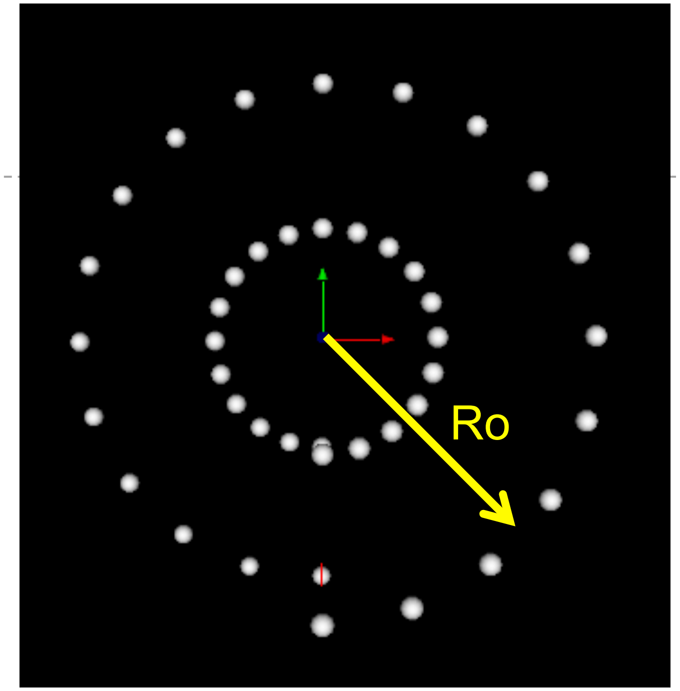



```
from vtk import *

pts = vtkPoints()
…
  pts.InsertPoint(i, x, y, z)   # e.g. pts on arc

pd = vtkPolyData()
pd.SetPoints(pts)

g = vtkGlyph3D()     # display spheres at pts
g.SetInput(pd)

pdm = vtkPolyDataMapper()
pdm.SetInputConnection(g.GetOutputPort())

arc1 = vtkActor()
arc1.SetMapper(pdm)
arc1.GetProperty().SetColor(1.0, 0.0, 0.0)
```

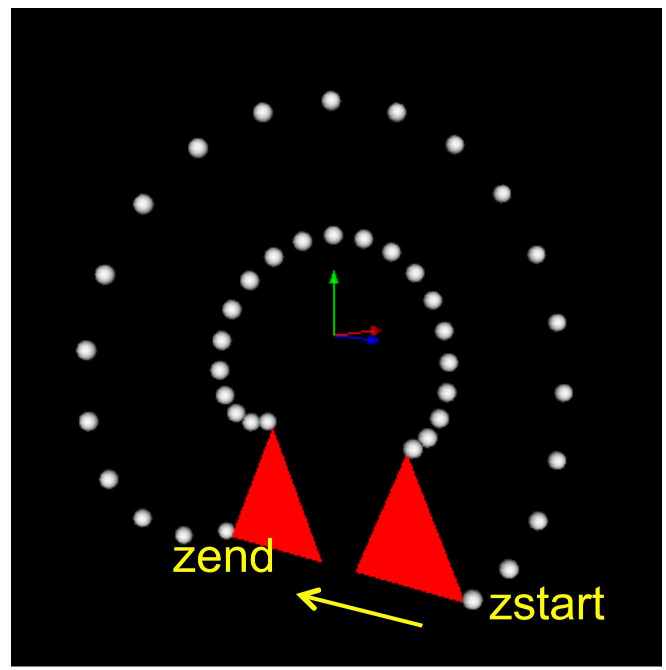
3D twisted annulus (with a cut)

```
t = 0.0
zdiff = zend - zstart
for i in range(0,npts):
    x = Ro * cos(theta)
    y = Ro * sin(theta)
    …
    z = zstart + zdiff * t
    t = t + tdel
```

Linear interpolation



Front, perspective view



Angled view

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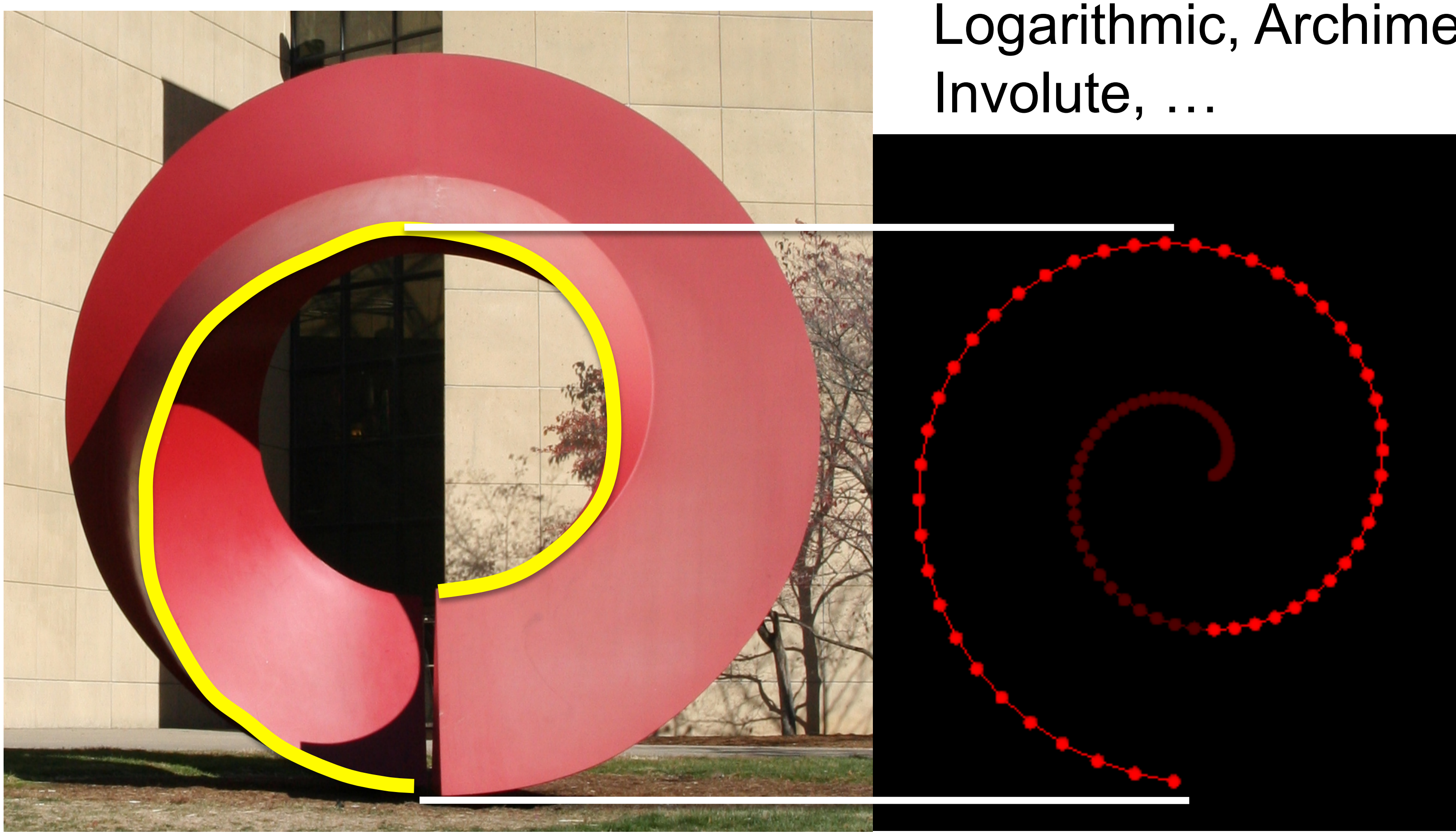
Profile curves: model as a spiral?

Logarithmic, Archimedean, Involute, …



x = aval*(math.cos(theta) + theta*math.sin(theta))
y = aval*(math.sin(theta) - theta*math.cos(theta))

Geometry and the Imagination

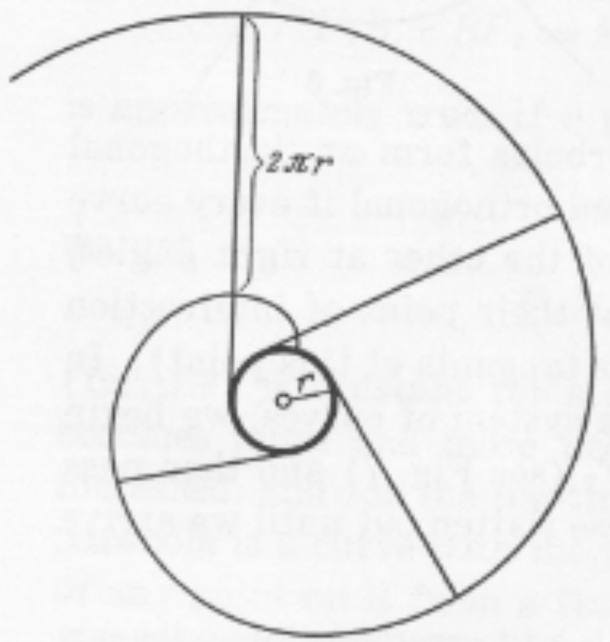
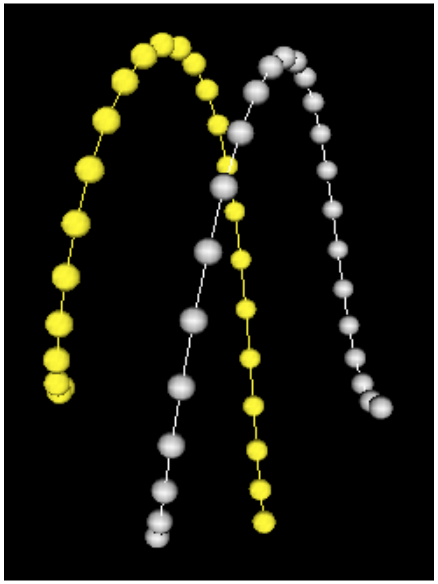
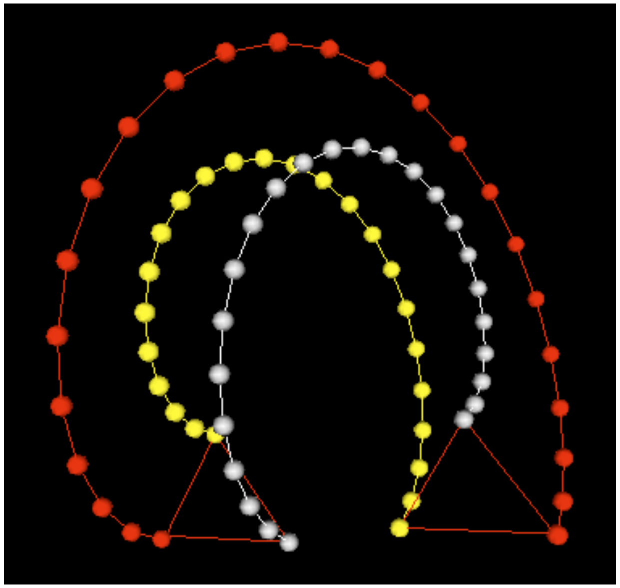
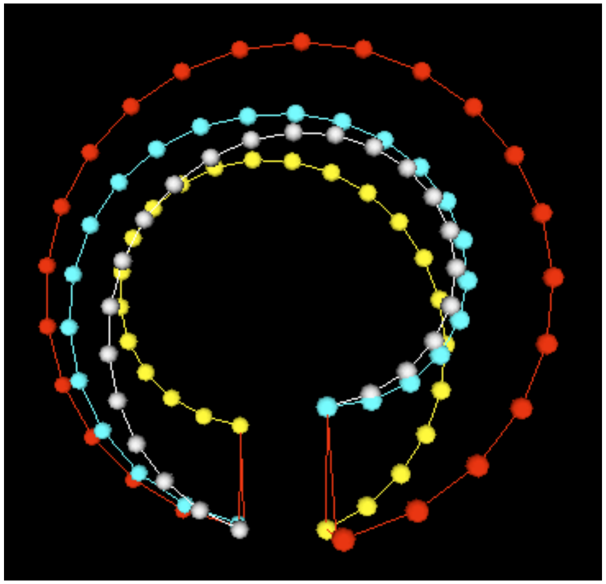


FIG. 8

…let us mention another thread construction that leads to an orthogonal net of curves. A thread is wound around a closed curve, say a circle. We consider the path traced by the free end when the stretched thread is unwound (see Fig. 8). The curve obtained in this way is called an "involute" of the circle, and is seen to be a spiral …

Spirals for profile curves…

…perhaps there's a better way?

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What about symmetry?

West

East

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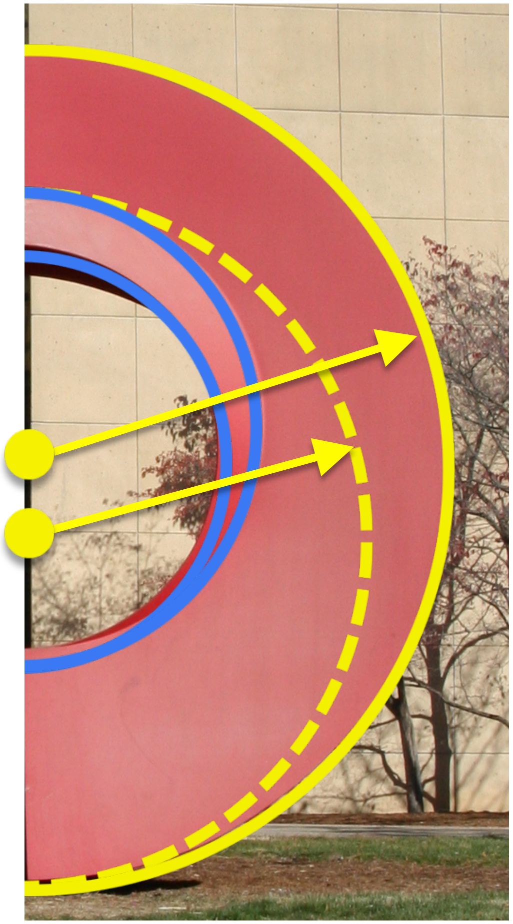
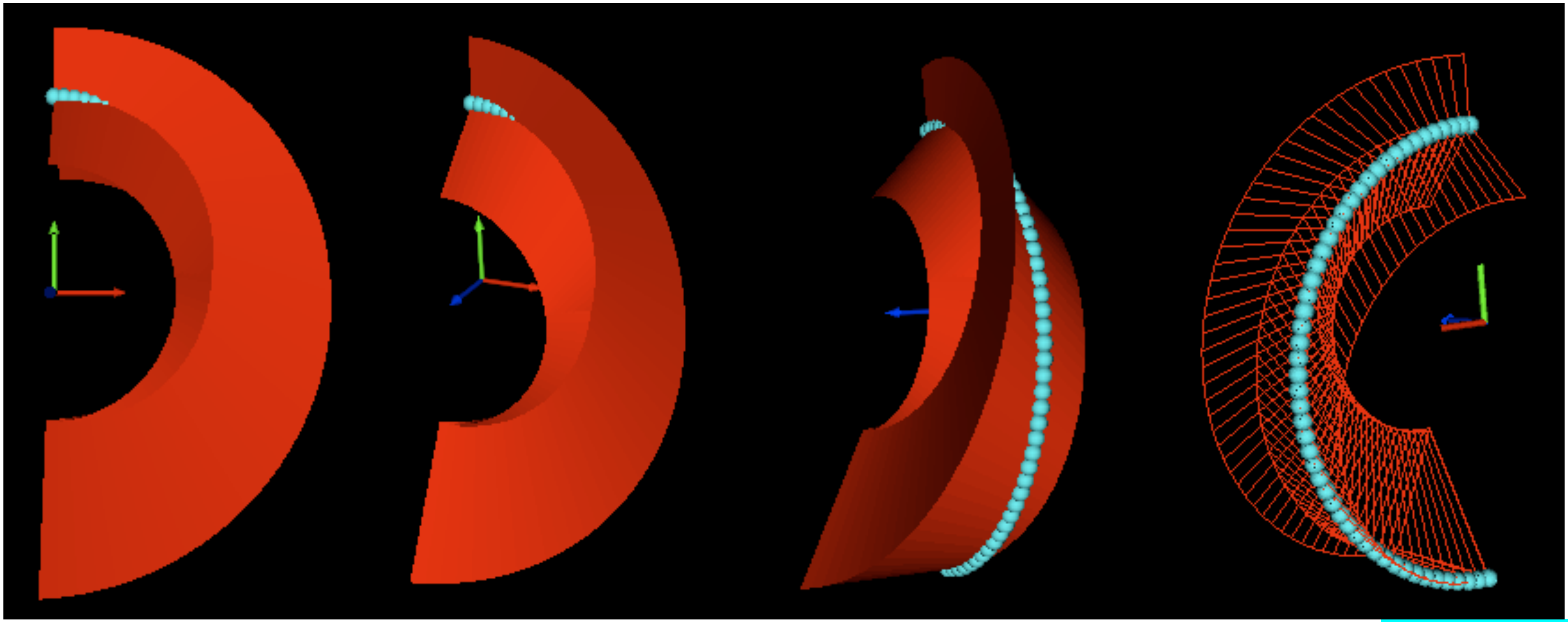
Symmetry: Mr. Perry calls it "Broken Symmetry"



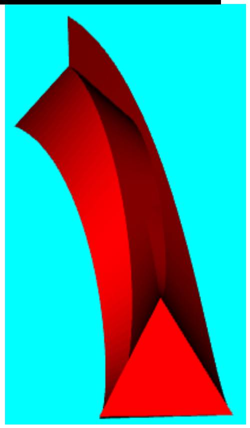
Can we model just half and then perform a symmetry transformation? If so, what kind? (→ matrix mult)

Now we model these profile curves as circular arcs. Each arc has a unique Center point: $(0,y_i)$ and unique Radius: R_i

Symmetry: model the right half



Circular arcs, interpolation, and
several model parameters.



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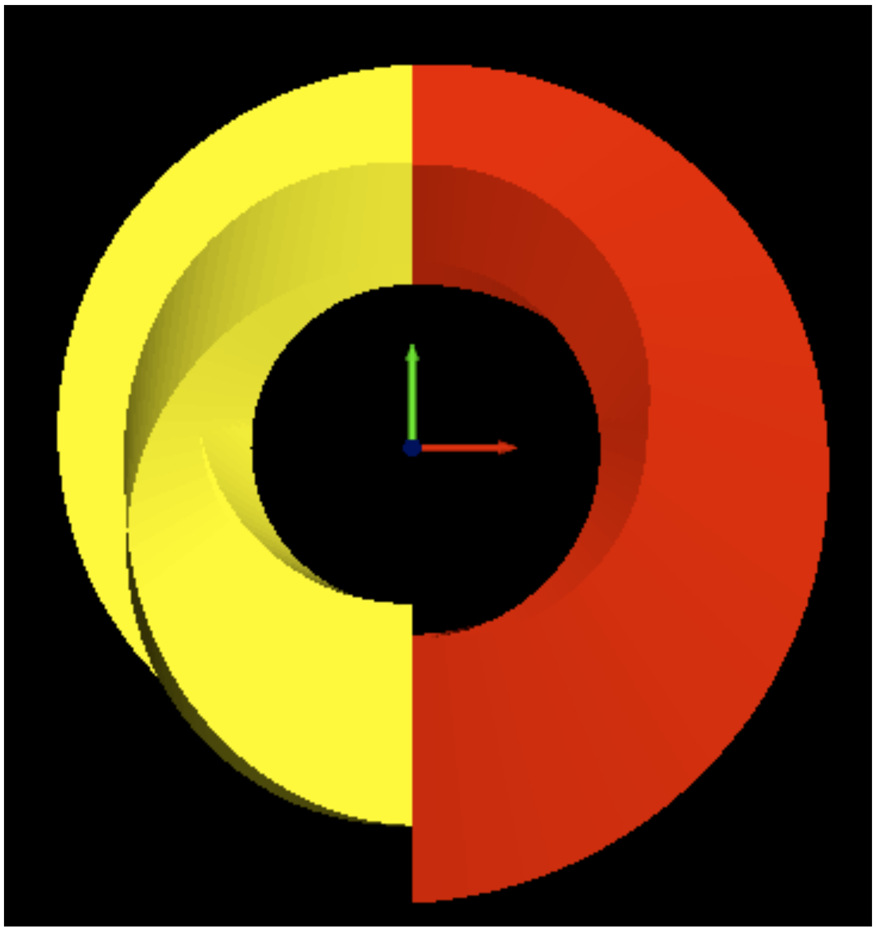
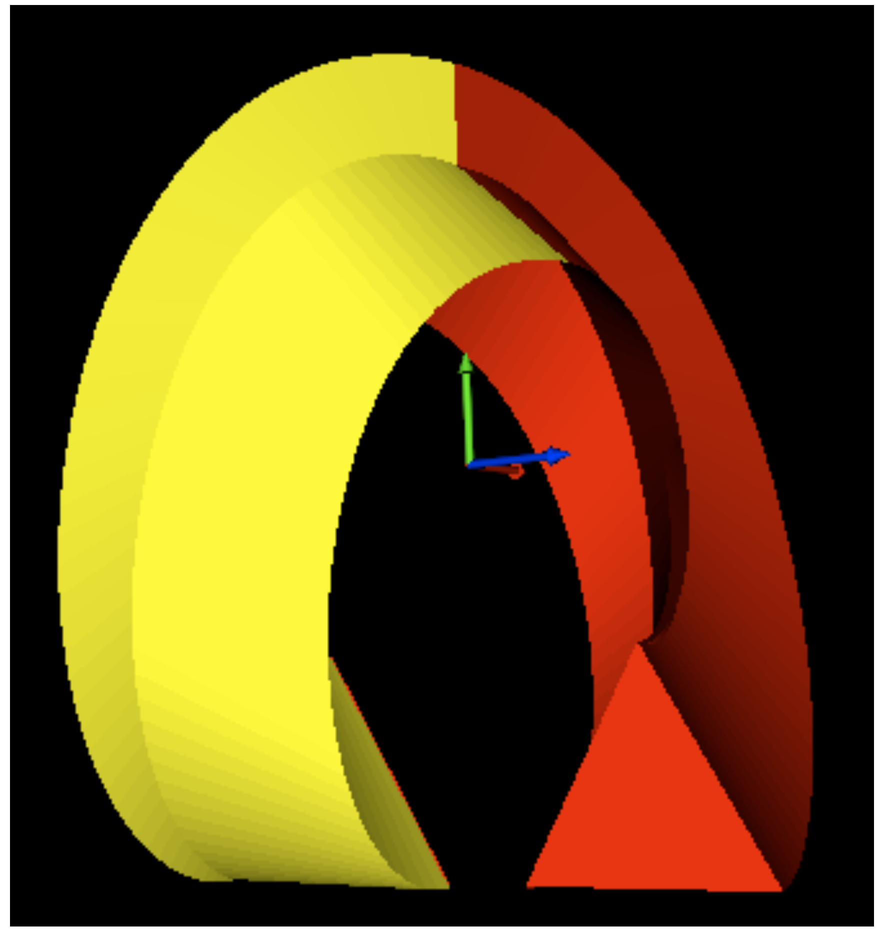
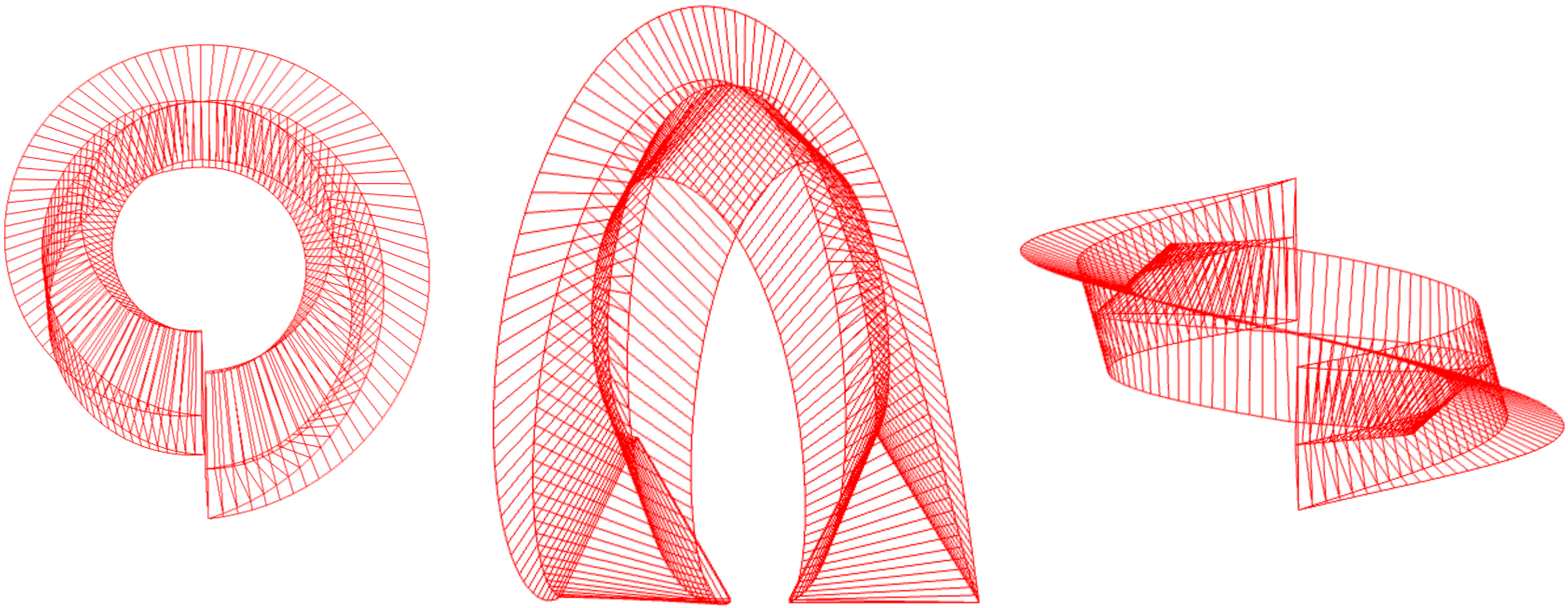
Symmetry transformation: RotY 180





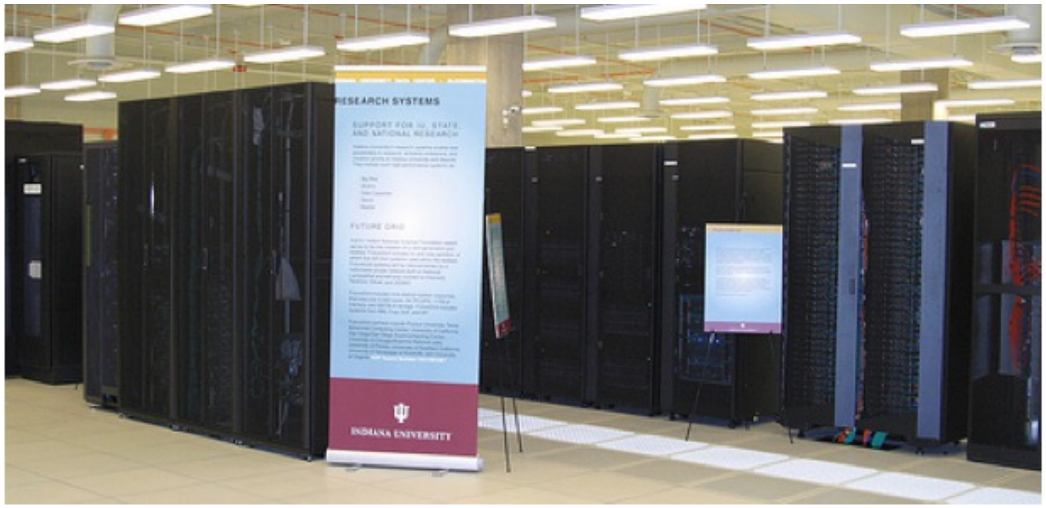
xform = vtkTransformPolyDataFilter()

(A) Final model in wireframe



Front Angled Top

Tangent #1: Computational Science



Tour IU's new Data Center.
Explain basics of cluster/parallel/research computing.
Submit jobs (on students' behalf) for a parameter study.

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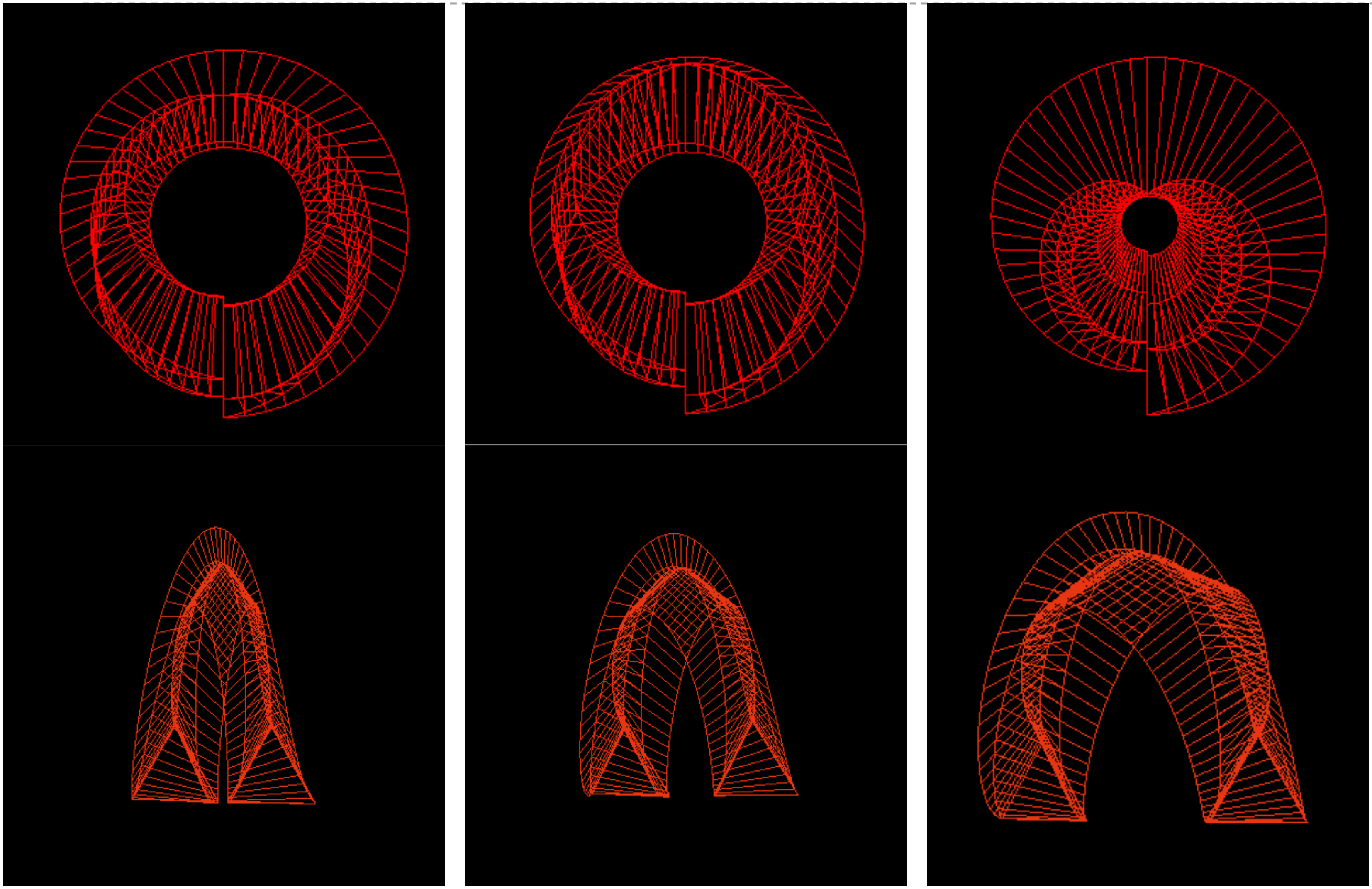
Montage of models: a parameter study



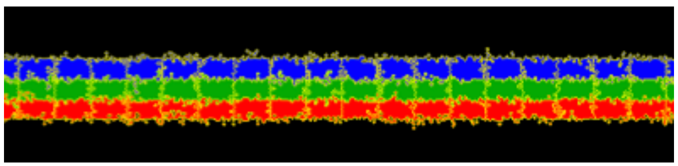
Radii

Separation

Tangent #2: Modeling concepts common in computational science

- Parameters (measure, calculate, explore)
- Create/discover patterns, shapes, structures

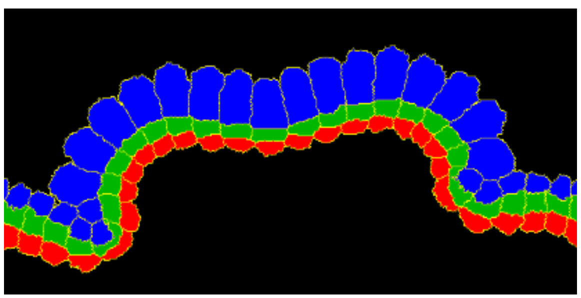
t_0



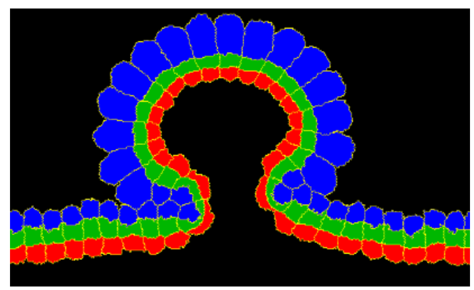
Biological modeling

e.g. retinal lens invagination (formation)

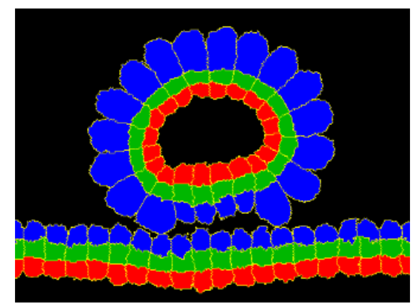
t_1



t_2



t_3



Credits: A. Shirinifard, J. Glazier (IU); M. Rountree (EPA)

compucell3d.org

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Conclusions

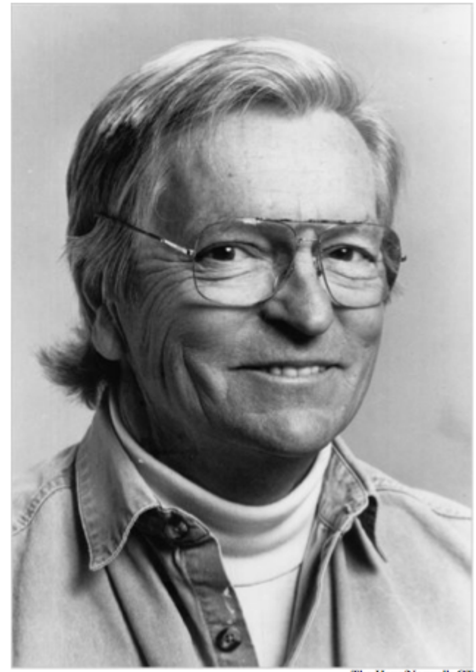
- Students lead incredibly busy lives; fitting in yet another extra-curricular is challenging
- … enjoyed the field trip and sketching
- … liked interactive 3D viewing of "their" geometry
- … netbooks – underpowered, but convenient
- … willing/able to do homework 2D plots (matplotlib)

- Work with more (larger) groups of students
- Incorporate 3D printing, stereo viewing
- Consider modeling other types of art
- Funding

Future

Questions?

- A special thanks to Mr. Perry, whose art was the inspiration for this project. He passed away on Feb 8, 2011.



The Hour/Norwalk CT